Introductory Laboratory Course

E1: Wheatstone Bridge

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Contents

1 Physical background and assignments

The Wheatstone Bridge is an electrical circuit that allows accurate measurement of electrical resistances. The experiment's manual mandates that, in order to corroborate the experimental results, Kirchhoff's circuit laws should be used as a way to theoretically determine the resistance of all circuits tested.

The first law is based on the conservation of charge in electrical circuits: it states that all electrical charges flowing into a particular junction of conductors (wires) have to leave that junction since a wire cannot hold electrical charge. Mathematically, this is expressed as $\sum I_i = 0$.

The second law states that the sum of voltages in a particular loop is zero: $\sum U_i = \sum R_i \cdot L_i$. The difference of potential between two points in an electrical circuit is independent from the particular path chosen between them, following a closed loop therefore leads back to the initial potential. The second law therefore follows from the conservation of energy.

In this experiment, Kirchhoff's laws and Ohm's law $U = R \cdot I$ are used to find formula (1.10) [Green Book, p. 3] as well as to determine the theoretical resistance of a particular combination of resistors through formulas (1.5) and (1.6) as described in the manual.

Other fundamental physical laws used in this experiment, as well as the specific tasks given to the experimenter, are laid out in detail in the manual [Green Book, pp. 1-5].

2 Data and data analysis

2.1 Resistors a, b, and c

To determine the resistance of the three single resistors used in this experiment, six measurements were taken each. The nominal resistances of a, b, and c were recorded using the colour code on each resistor. These values can provide a point of comparison for the experimental results found with the help of the Wheatstone Bridge:

$$a_n = (12 \pm 0,12)\ \Omega$$
$$b_n = (68 \pm 0,68)\ \Omega$$
$$c_n = (120 \pm 1,20)\ \Omega$$

The normal resistance R_n [Green Book, pp. 3] was set as a combination of three adjustable precision resistors R_i where i is the increment:

$$R_{100} = 100\ \Omega ... 1000\ \Omega \pm 0,2\%$$
$$R_{10} = 10\ \Omega ... 100\ \Omega \pm 0,2\%$$
$$R_1 = 1\ \Omega ... 10\ \Omega \pm 0,5\%$$

Since the three precision resistors had different uncertainties their exact configuration was recorded for each measurement as $\sum R_i$, and the total uncertainty u_{R_n} calculated as the sum of their absolute errors. The total length of the bridge was $l = 1\ m$ with an error small enough to ignore. The reading error of x was estimated to be $u_x = 0,001\ m$.

In the circuit of a Wheatstone Bridge, the unknown resistance R_x can be calculated as:

$$R_x = R_N \frac{x}{l - x} \tag{1}$$

while the corresponding u_{R_x} is found with the help of the laws of error propagation [Blue Book, p. 36]:

$$\frac{u_{R_x}}{R_x} = \sqrt{(\partial_x R_x \cdot u_x)^2 + (\partial_{R_N} R_x \cdot u_{R_N})^2} = \sqrt{\left(\frac{u_{R_N}}{R_N}\right)^2 + \left(\frac{l \cdot u_x}{x(l-x)}\right)^2} \tag{2}$$

Using (1) and (2), the following values were found for a, b, and c:

$$a = (11,92 \pm 0,03)\ \Omega$$
$$b = (67,61 \pm 0,17)\ \Omega$$
$$c = (119,53 \pm 0,30)\ \Omega$$

2.2 Five combinations of (a, b, c)

The resistors a, b, and c were combined in five different ways and the resistance of each combination was measured with the help of the Wheatstone Bridge and, in addition, calculated theoretically according to Kirchhoff's circuit laws.

In the following notation, a "$-$" signifies a series of resistors, whereas "$\|$" denotes a parallel combination. The theoretical resistances of the three basic types of combinations can be calculated with the help of Kirchhoff's circuit laws and Ohm's law as represented by formulas (1.5) and (1.6) [Green Book, pp. 2].

A series of three resistors, type $R_1 - R_2 - R_3$, has a total resistance R_t of:

$$R_{t,s3} = R_1 + R_2 + R_3$$

The resistance of a series of one resistor and one parallel combination of two resistors, type $R_1 - (R_2 \parallel R_3)$, is:

$$R_{t,p2} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = R_1 + \frac{R_2 R_3}{R_3 + R_2}$$

Finally, a parallel circuit of one resistor and one series of two resistors, type $R_1 \parallel (R_2 - R_3)$, can be calculated as:

$$R_{t,p3} = \left(\frac{1}{R_1} + \frac{1}{R_2 + R_3}\right)^{-1} = \frac{R_1 + R_2 + R_3}{R_1(R_2 + R_3)}$$

The following table summarizes the results. The *experimental* entry was calculated from experimental data based on formula (1). The *theoretical/exp* entry gives the resistance of the circuit as calculated with Kirchhoff's circuit laws based on the experimentally found values of a, b and c. Finally, the *theoretical/nom* values are included for comparison: they calculate the resistance of each circuit with the help of Kirchhoff's laws but are based on the nominal values a_n, b_n and c_n.

Each R_x and its error was calculated as a weighed average of the six measurements taken, using formulas (55), (56) and (57) in [Blue Book, p. 47].

Table 1: Resistor configurations

	Experimental		Theor./exp		Theor./nom	
	R_x [Ω]	u_{R_x} [Ω]	R_x [Ω]	u_{R_x} [Ω]	R_x [Ω]	u_{R_x} [Ω]
$a - b - c$	199,10	0,54	199,05	0,22	200,00	2,00
$c - (a \parallel b)$	129,81	0,35	129,66	0,13	130,20	2,00
$a - (b \parallel c)$	55,07	0,15	55,10	0,04	55,40	0,34
$(a - b) \parallel c$	47,71	0,12	47,75	0,03	48,00	0,10
$(b - c) \parallel a$	11,19	0,03	11,21	0,01	11,28	0,43

2.3 Resistance cube

The resistance cube as pictured in the manual [Green Book, Figure 1.2, p. 2] was measured across three different axes: $\overline{AB}$ (edge of the cube), $\overline{AC}$ (diagonal across one side), and $\overline{AD}$ (body diagonal). To provide a point of comparison, R_x of the different axes was also measured by multimeter. According to Kirchhoff's laws, the installed resistors' resistance R can then easily be calculated from the experimentally measured value of R_x.

Table 2: Resistance cube, different axes

	Experimental				Multimeter			
	R_x [Ω]	u_{R_x} [Ω]	R_i [Ω]	u_{R_i} [Ω]	R_x [Ω]	u_{R_x} [Ω]	R_i [Ω]	u_{R_i} [Ω]
$\overline{AB}$	22,6	0,1	38,8	0,2	22,6	0,7	38,7	1,2
	22,5	0,1	38,6	0,2				
$\overline{AC}$	29,0	0,2	38,7	0,3	29,2	0,7	38,9	1,3
	29,6	0,2	39,5	0,4				
$\overline{AD}$	32,3	0,2	38,7	0,3	32,4	0,8	38,9	1,3
	32,2	0,2	38,7	0,3				

In order to find R, it is necessary to determine how R_i and R_x relate. This has been done by simplifying the schematics of the cube as shown in Figures 1, 2, and 3 in the Annex:

For $\overline{AB}$, refer to Figure 1: if the cube's schematic is drawn out, four points of equal potential can be identified (cf. green circles in 1.a). Connecting those, an alternative schematic can be found (1.b). In this circuit, all pairs of parallel resistances are combined into a single resistance $R/2$, leading to schematic (1.c). Then, the two outer resistances and the bottom resistance are combined ($2R/2 + R = 2R$) so that a parallel circuit with the inner resistance can be calculated:

$$R_* = \left(\frac{1}{R/2} + \frac{1}{1/2R} \right)^{-1} = \frac{2}{5}R$$

Finally, the total resistance of the cube along $\overline{AB}$ can be determined from the last schematic (1.d) as a parallel connection between R and the three remaining resistances:

$$R_{AB} = \left(\frac{1}{R} + \frac{1}{1/2R + 2/5R + 1/2R} \right)^{-1} = \left(\frac{14}{24R} \right)^{-1} = \frac{7}{12}R$$

Similarly, it is possible to determine R for $\overline{AC}$. Consider Figure 2.a: the points of equal potential are marked by red and green circles. Using this information, and combining pairs of parallel resistors into single resistances, schematic (2.b) can be found. Subsequently, the series of R and $R/2$ resistances on the sides are calculated. In (2.c) symmetry mandates that the potential at both ends of the resistor in the centre is equal, which can thus be ignored. The final resistance of the cube can hence be determined with the help of the greatly simplified circuit (2.d):

$$R_{AC} = \left(\frac{1}{R/2} + \frac{1}{3/2R + 3/2R} \right)^{-1} = \frac{3}{4}R$$

Lastly, for $\overline{AD}$, R can be found as indicated in Figure 3: a simple alternative circuit can be drawn considering the three points of equal potential on either side of A and D. The remaining resistances in between these points are treated as a parallel combination of six resistors. Using this alternative layout, the resistance of the cube along $\overline{AD}$ is:

$$R_{AD} = \left(\frac{1}{3R} + \frac{1}{6R} + \frac{1}{3R} \right)^{-1} = \frac{5}{6}R$$

In order to determine a single, final value for R, all six individual R_i were combined into a weighed average R_{exp} according to formulas (55), (56) and (57) [Blue Book, p. 47]. Similar steps were taken with regard to those values obtained by multimeter, and leading to the weighed average R_{mm}:

$$R_{exp} = (38,8 \pm 0,1)\ \Omega$$
$$R_{mm} = (38,9 \pm 0,5)\ \Omega$$

3 Error analysis and results

Given that in all cases the experimental results for R_x and R agree with those values obtained through theoretical calculations and measurements by multimeter, grave mistakes on the part of the experimenters are unlikely. In fact, the relative deviation within the sets of values given in Table 1 does at no point exceed 0,7%, the relative deviation between R_{exp} and R_{mm} is even lower, highlighting the consistency of our results.

It should be noted that the values that stem from experimental data have significantly lower uncertainties than the nominal values and even those measured by multimeter: the Wheatstone Bridge is thus able to provide a relatively easy and cheap alternative to other ways of measuring electrical resistances, e.g. digital equipment.

A simple way to increase the accuracy of this experiment is to use higher quality components in order to minimize impedance in lead wires and connectors. In addition, more accurate precision resistances R_N could be considered since u_{R_N} is by far the largest contributor to the final uncertainty of R_x. Consequently, it is important to always use the highest available resistor instead of those with lower increments in order to minimize the relative error: a tentative measurement taken with a 0-5-10 configuration instead of the standard 0-6-0 increased uncertainty by more than 20%.

References

[Blue Book] Müller, U. *Einführung in die Messung, Auswertung und Darstellung experimenteller Ergebnisse in der Physik.* 2007.

[Green Book] Müller, U. *Physikalisches Grundpraktikum. Elektrodynamik und Optik.* 2010.

A Appendix

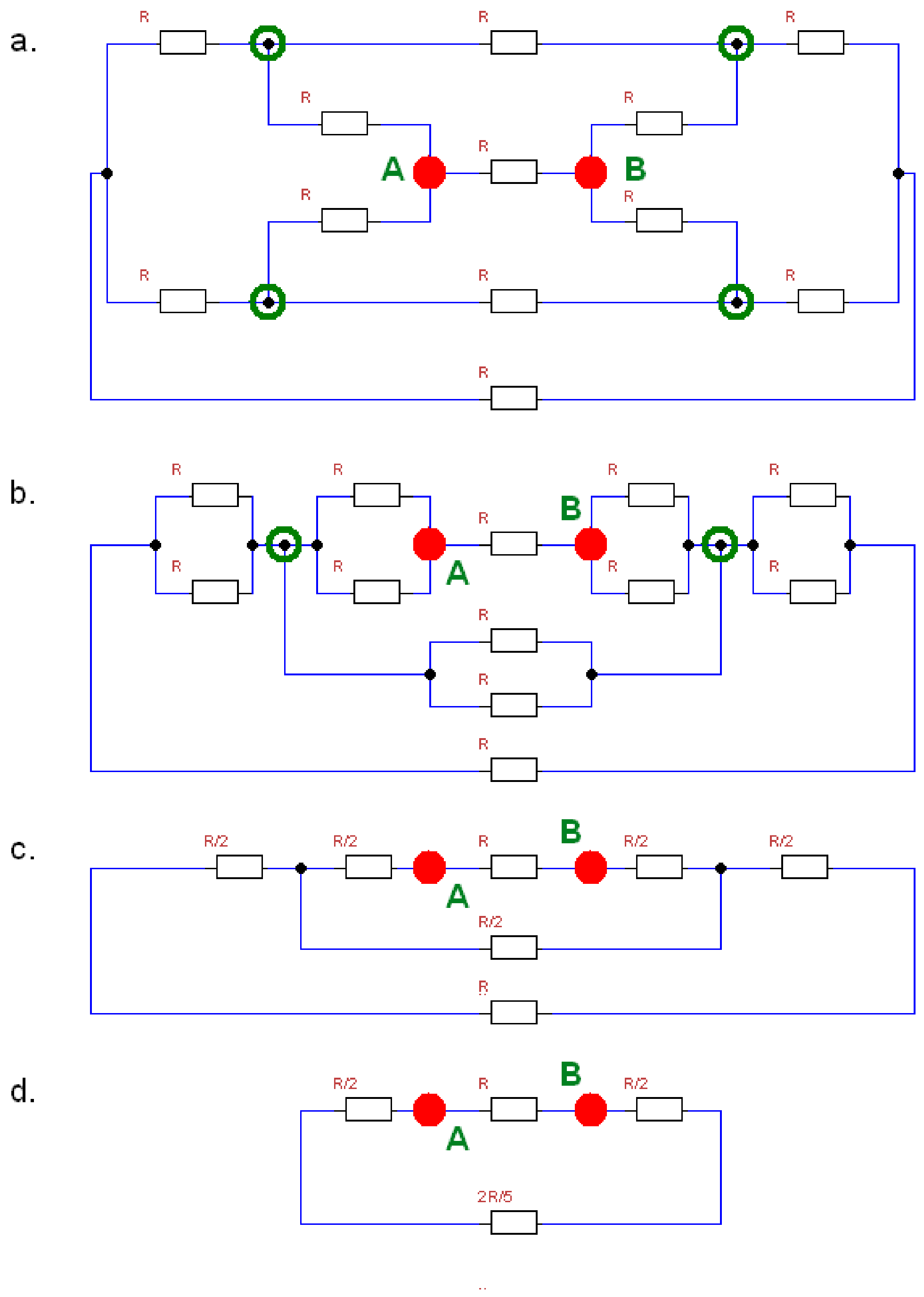

Figure 1: On the calculation of the resistance of $\overline{AB}$

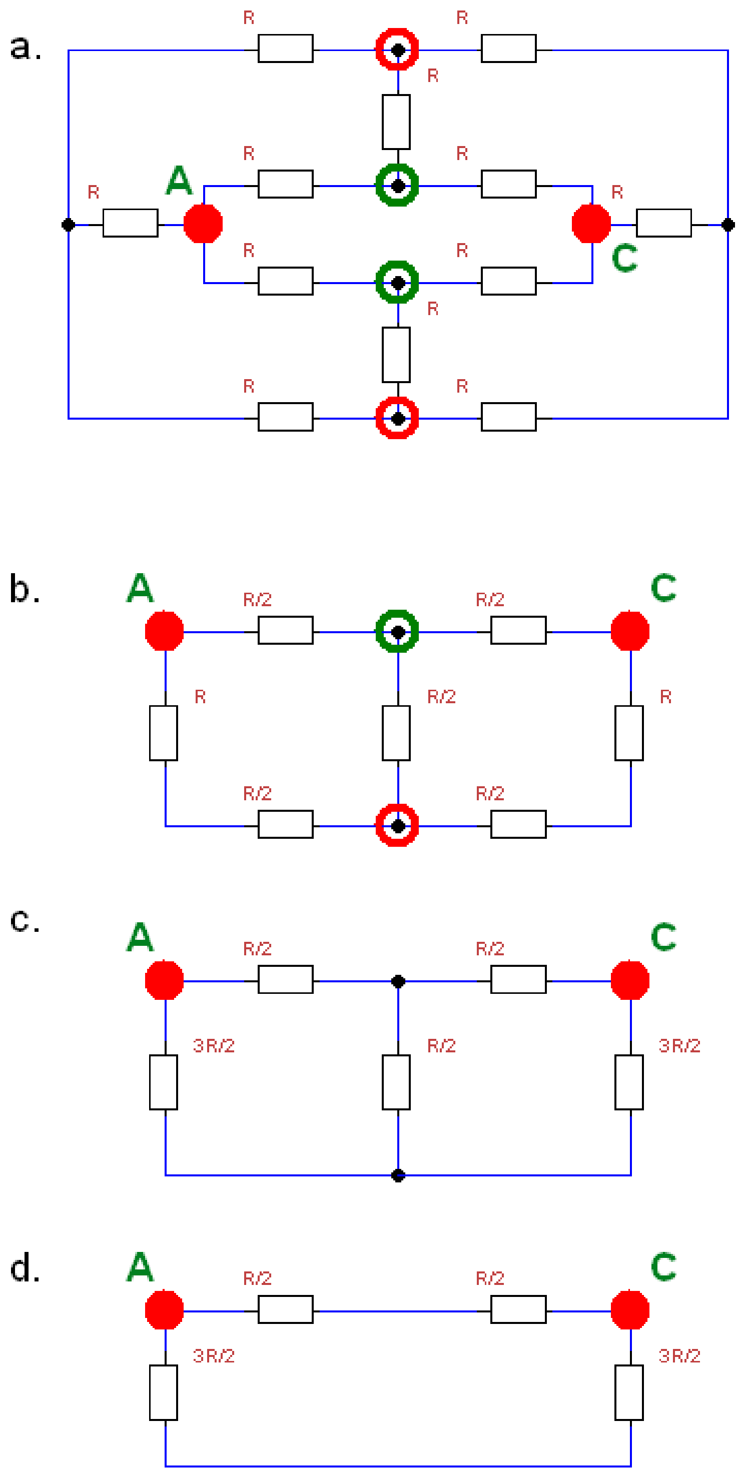

Figure 2: On the calculation of the resistance of $\overline{AC}$

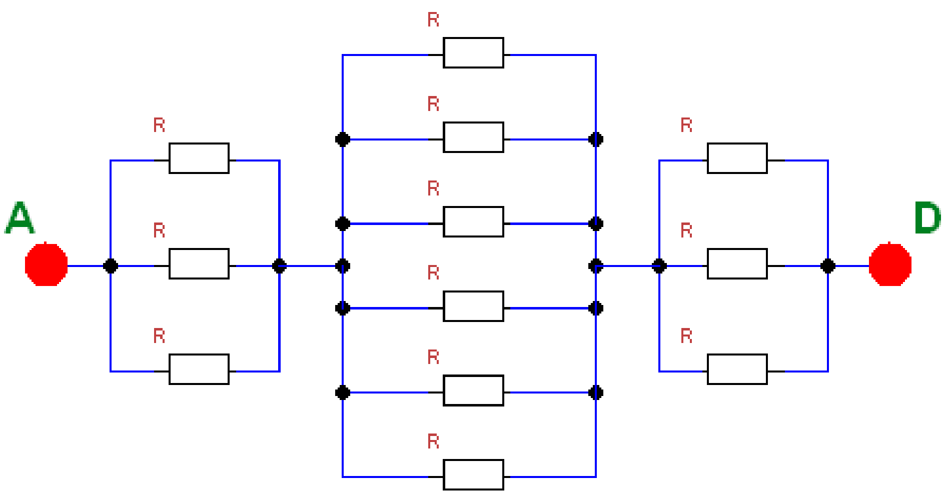

Figure 3: On the calculation of the resistance of $\overline{AD}$